Was the Titius–Bode Series Dictated by the Minimum Energy States of the Generic Solar Plasma?

DANIEL R. WELLS

Abstract—**It has been shown in detail elsewhere [6] that the Bode numbers and measured velocity ratios of the planets are accurately predicted by the eigenvalues of the Euler–Lagrange equations resulting from the variation of the free energy of the generic plasma that formed the Sun and planets. This theory is reviewed and extended to show that the equations make accurate predictions for all the major planets out to and including Pluto. The semimajor axes and velocity ratios of Pluto and Neptune are predicted exactly. The Bode numbers are shown in Table I to correspond to the roots of the first-order Bessel functions. The extrema of the roots of the zeroth-order Bessel function predict the ratios of the measured planetary velocities almost without error for the outer planets. Both sets of roots correspond to the same eigenvalue solution of the force-free equation. The eigenvalues are set by the initial energy input to the plasma nebula. Both the Titius–Bode series and Kepler's harmonic law are predicted by the "relaxed state solution" of the free-energy equation for the generic plasma that formed the Sun and planets. Newton's law of gravitation is not used in the calculations. The solution makes exact predictions for the outer planets where the Titius–Bode series fails completely.**

I. Introduction

FOR MORE THAN 30 years it has been understood that the lowest energy state of a finite volume of plasma subject to the appropriate boundary condition corresponds to a force-free collinear flow structure [2]. This early work was applied to laboratory systems by Wells [8], and later by Taylor [4] and others [3]. The author considered the problem of applying these methods to the morphology of the solar system [6]. He was able to predict the Bode numbers and show that they corresponded to the roots of the Bessel function solutions of the appropriate force-free equation. We outline the theory in Section II. In Section III the theory is extended to a prediction of the semimajor axes and velocity ratios of the outer planets not given in the earlier papers.

II. How was the Morphology of the Solar System Set?

Assume a major disturbance in the primordial gas of the galaxy. The compressed gas forms a cylindrical volume of plasma which is moving through the background plasma and rotating with a finite angular velocity (Fig. 2). Assume that there is a finite magnetic field present during the formation of the plasmoid.

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The author is with the Department of Physics, University of Miami, Coral Gables, FL 33124.
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As this mass of plasma propagates through the surrounding gas, it loses energy by accelerating the surrounding plasma. The cylinder will lose energy and settle down to a minimum-energy "relaxed" state, a force-free collinear cylindrical structure. It is shown in detail elsewhere [5]–[7] that the resulting "field equations" for the flow are given by

$$\nabla \times \boldsymbol{B} = k\boldsymbol{B} \tag{1}$$

$$v = \pm \left[\frac{\gamma - 1}{\gamma - 2}\right] \frac{\boldsymbol{B}}{(\mu_0 \rho)^{1/2}} \tag{2}$$

where

$\rho \overset{\text{def}}{=}$ fluid density,
$\boldsymbol{B} \overset{\text{def}}{=}$ magnetic induction field,
$v \overset{\text{def}}{=}$ velocity of the center of mass of a fluid element,
$\gamma \overset{\text{def}}{=}$ ratio of specific heats of the gas.

A *pseudoplane* solution to the force-free equation (1) is given by Bjorgum and Godal [1] and is shown to be [6], [1]:

$$B_r = -k^2\tilde{a}\,\frac{J_1(kr)}{kr}\sin\theta$$

$$B_\theta = k^2\tilde{a}\left\{\frac{J_1(kr)}{kr} - J_0(kr)\right\}\cos\theta$$

$$B_z = k^2\tilde{a}J_1(kr)\cos\theta$$

$$B^2 = \tilde{a}^2k^4\left\{\left(\frac{J_1}{kr}\right)^2 - \frac{2J_0J_1}{kr}\cos^2\theta + (J_0^2 + J_1^1)\cos^2\theta\right\}$$

where B_r, B_θ, and B_z are the magnetic induction components in the striated rings of the gas cylinder, and

$$\tilde{a} = l\theta + k_2 z, \qquad k^2 = k_1^2 + k_2^2$$

where l, k_1, and k_2 are constants supplied by the boundary conditions [1]. Fig. 1 shows a plot of J_1 and J_0 with the functions scaled to the geometry of the solar system. We observe that for $J_1 = 0$,

$$B^2 \sim J_0^2$$

$$B_\theta \sim J_0.$$

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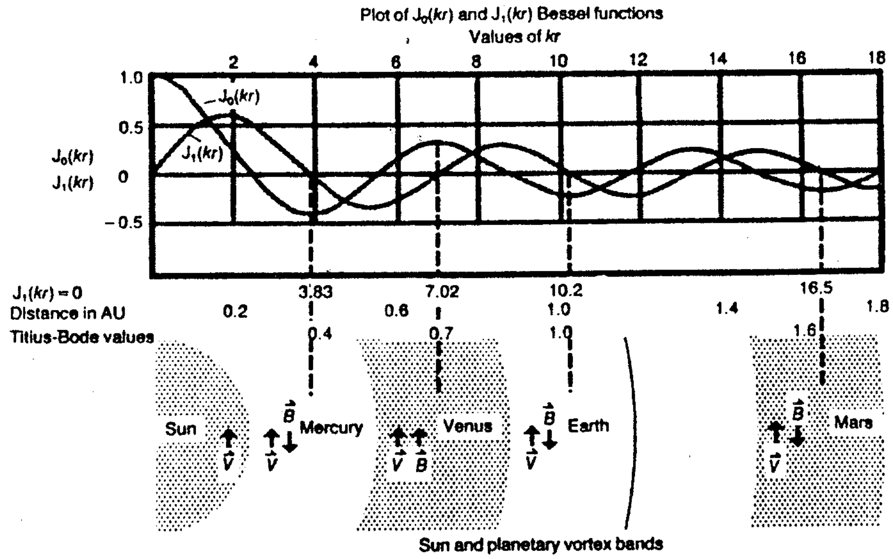

Fig. 1. Agreement of Bessel functions with Titius–Bode and measured planetary distances. A plot of the J_1 and J_0 Bessel functions is scaled here to the geometry of the solar system. The values shown here are also summarized in Table I. Corotational vortex bands (planets) must alternate with contrarotational ones. In the first case, orbital motion (vector V) and the magnetic field (vector $\boldsymbol{B}$) are aligned; in the latter, they are opposed. The missing planet just beyond Earth (indicated by a curved line) would have to have been contrarotational. Note that the orbital motion of all planets, and also the rotational motion of the Sun, are in the same direction, as they are in fact.

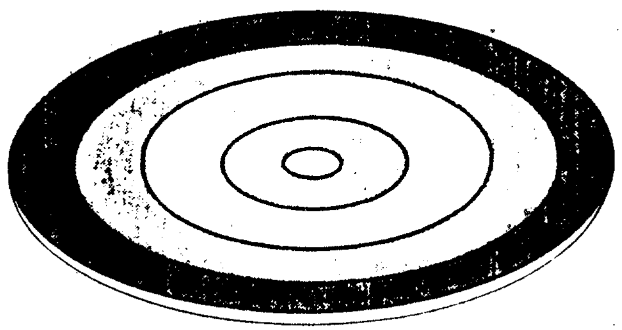

Fig. 2. Planetary vortex bands. At this stage the concentric vortex cylinders have collapsed into a disc of concentric bands. The matter in each band will subsequently form a ring and then a planet.

This maximizes the magnetic and kinetic energy at the origin. In the cylindrical structure formed by the supernova explosion, the first root corresponds to the structure of the star at the center of the hypothetical solar system, and the second root corresponds to a ring of gas just outside the star. The corresponding flow velocities in the rings is given by (2). The geometry of the configuration is shown in Fig. 2. The signs reverse for every other ring (corotational and contrarotational) so that the azimuthal velocities are all prograde. The azimuthal velocity of the gas in each ring has a direct relationship to the velocities of the planets as they exist today. An examination of Table I shows that the Bode numbers of the planets out to Jupiter are predicted by the roots of the equations describing the "relaxed state" of the primordial gas. Comparison of the measured velocity ratios with the ratios of the extrema of $J_0(kr)$ show very close agreement.

III. The Outer Planets

For the outer planets, the Bode series fails completely for Neptune and Pluto, but the plasma solutions, the Bessel functions, ... early papers on this subject took the Bessel function solutions out to and including the planet Jupiter [6], [7]. Allen observed [9] that if the asymptotic expansions of $J_1(kr)$ and $J_0(kr)$ were carried out, the theory could be checked all the way out to and including Pluto. Beyond the 50th root, $J_1(kv)$ increases by approximately 3.14 between roots. $J_0(kr)$ is given by

$$J_0(kr) \simeq \left(\frac{2}{(\pi(kr))}\right)^{1/2}.$$

The corresponding normalized (kr) values are shown in Table I. The predicted ratios of the successive peak velocities of the gas in the rings check the measured velocity ratios of the inner planets within a few percent. The velocity ratios for Uranus, Neptune, and Pluto are exact. The relaxed state of the generic plasma predicts both the Bode number series and Kepler's harmonic law:

$$p^2 = a^3$$

where

$p \overset{\text{def}}{=}$ period of the planet,
$a =$ average radius of the planet.

It is suggested that the rings of gas in the planet structure "roll up" azimuthally to form balls of gas that eventually evolve into the planets (Fig. 3) [6]. The rollup of vortex rings to form balls of gas is a well-known phenomenon which has been observed in laboratory experiments [6]. This is illustrated in Fig. 3.

IV. Discussion

The theory developed in this paper not only accurately predicts the semimajor axes of the nine major planets and their velocity ratios without invoking Newton's law of

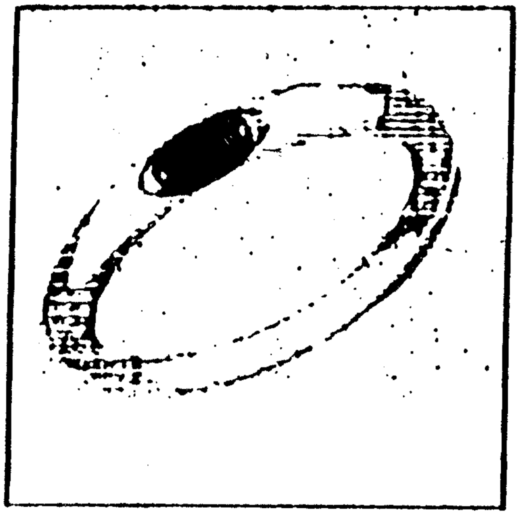

Fig. 3. Toroidal eigenmode contractions.

TABLE I
BODE NUMBERS

Planet	Root n	Bode numbers	Normalized (kr_n)values	Measured AU	% Error	Extremum $J_0(kr_n)$	Velocity Ratio	Extremum Ratio	% Error
Mercury	1	.4	.38	.39	-2.6	0.4028			
							1.37	1.34	-2.2
Venus	2	.7	.69	.72	-4.2	0.3001			
							1.18	1.20	+1.7
Earth	3	1.0	1.0	1.0	–	0.2497			
							1.24	1.27	+2.4
Mars	5	1.6	1.62	1.52	-6.6	0.1965			
							1.86	1.76	-5.4
Jupiter	16	5.2	5.02	5.20	-3.5	0.1117			
							1.34	1.39	+3.7
Saturn	31	10.2	9.62	9.54	+.80	0.0805			
							1.42	1.41	-0.7
Uranus	62	19.6	19.17	19.2	-.16	0.0571			
							1.25	1.25	0.00
Neptune	97	38.8	29.95	30.1	-.49	0.0457			
							1.15	1.15	0.00
Pluto	128	77.2	39.49	39.5	-.03	0.0398			
Asteroids	11		3.08(Avg)	3.35(Avg)	-8.1				

:avitation (inverse-square law), but also hints at other ıenomena associated with the morphology of the sys- m. These may be listed as follows:

1) A planet is predicted at 1.3 AU. No such planet ex- :s today. It is suggested that the missing planet suffered catastrophe either in the birthing process or at a later ne and that the residue is our moon.

2) There are 11 rings possible between Mars and Ju- ter. The asteroid belt is located in this region. Table I licates that the average predicted semimajor axes of ese rings correspond to the average semimajor axes of e principal asteroids to an accuracy of -8.1 percent.

3) The errors in the predicted semimajor axes of Mars d Jupiter and the velocity ratio of Mars to Jupiter is ge compared to the values for the outer planets. This ıy be caused by a disruption due to whatever mecha- ım violently formed the asteroids.

4) Both the predicted semimajor axes and velocity ra- tios are almost exact for the outer planets (Uranus, Neptune, Pluto).

5) The exact match for the predicted semimajor axes of Uranus, Neptune, and Pluto is especially interesting. For a fixed k (see (1)), i.e., a fixed eigenvalue for the force-free equation, the spacing of the roots as a function of radius is fixed. If one now normalizes the system so that the third root has an argument equal to 1 AU (the semimajor axes of Earth), then there is a high-order root (62 for Uranus, 97 for Neptune, 128 for Pluto) corresponding exactly to the semimajor axes of Uranus, Neptune, and Pluto. The extrema of $J_0(kr)$ also accurately predict the measured velocity ratios of these planets.

6) It is suggested that the rings corresponding to the roots that do not match any existing planets were either pulled into the planetary rings or formed gas balls that were not stable. The gas was then lost to the system or later formed planetisimals that account for the high level of meteoritic activity in the early solar system.

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Daniel R. Wells received the Ph.D. degree in plasma physics at the Stevens Institute of Technology, Hoboken, NJ, in 1963.

He is now Professor of Physics at the University of Miami, Coral Gables, FL. His theory of minimum-energy configurations for magnetic confinement of hot, thermonuclear plasmas has become the basis for fusion experiments involving spheromaks, compact tori, and reverse-field pinches throughout the world.