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SHIFTS OF SPECTRAL LINES CAUSED BY SCATTERING FROM FLUCTUATING RANDOM MEDIA

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ABSTRACT

\ model scatterer is introduced, whose dielectric response function is a random function of space and time 1 which produces frequency shifts of spectral lines that imitate the Doppler effect in its main features. The ;sible relevance of this effect to the origin of discrepancies observed in some quasar spectra is discussed.
)ject headings: galaxies: redshifts — quasars — radiation mechanisms

I. INTRODUCTION

t few years several new closely related processes liscovered that can generate frequency shifts of s. This development followed the theoretical pre- ; by Wolf (1986) and since then verified by experi- s and Faklis 1987) that, contrary to a commonly he spectrum of light is, in general, not invariant on in free space. For radiation from primary sources, changes are induced by correlations between fluc- the source distribution at different points in the n (Wolf 1987*a*, *b*; see also James and Wolf 1989); ı from secondary sources, such as an illuminated changes are induced by the correlations between tuations at pairs of points in the aperture plane Dacic and Wolf 1988; Gamliel 1989). In scattering ; changes in the spectrum are induced by corre- en fluctuating response functions of the scattering ., its dielectric susceptibility, either at different scatterer, when the frequency-dependent macro- nse is time-independent (Wolf, Foley, and Gori : different spacetime points, when it is time- 'oley and Wolf 1989). In all these cases the spectral the consequence of correlations involving an variable characterizing the source, the field, or the a scatterer. With appropriate correlations, the manifested as frequency shifts of spectral lines. ity has been predicted theoretically (Wolf 1987*a*, y, and Gori 1989; Foley and Wolf 1989) and has ed by a number of laboratory experiments using controllable coherence properties, both optically Morris 1988; Gori *et al.* 1988; Indebetouw 1989), ustically (Bocko, Douglass, and Knox 1987) for e shifts are induced by source correlations.

tly it was predicted (Wolf 1989*a*, *b*) that the fre- s induced by scattering from time-dependent a with suitable correlation properties may imitate ift of any magnitude, even though the source, the the observer are all at rest with respect to each ıs therefore possible, in principle, that this effect te toward the redshifts observed in the spectra of mical objects. In this connection we recall that nding controversy over the interpretation of quasar redshifts continues despite the pronouncement that "the 'evidence' for non-cosmological redshifts is a collection of unrelated curiosities having no predictive power" (Weedman 1986, p. 37). The contrary view was ably argued by Arp (1987). Even Hubble (1936) questioned the validity, in all cases, of the velocity interpretation. Most non-Doppler hypotheses proposed in the past violated established physical laws in some way, e.g., the principle of conservation of momentum and energy.

It is important to appreciate that there are two quite distinct aspects to the redshift controversy, which have not always been kept in focus in the lengthy dispute, namely, the questions (1) whether most quasars are at cosmological distances and (2) whether *some* quasars are associated with objects of lower redshifts (e.g., Markarian 205 with NGC 4319). We wish to make it quite clear at the outset that, in this paper, we are not questioning whether most quasars are at cosmological distances. However, we show that scattering from suitably correlated fluctuating random media such as those discussed in this paper provides a possible additional mechanism for generating redshifts.

In the present paper we consider an explicit form for a correlation function of the dielectric susceptibility of a fluctuating random scattering medium which will generate frequency shifts that are essentially indistinguishable from those that might be caused by the motion of a source relative to the observer or by gravitation; and we show that the usual quasar models imply characteristic anisotropies which are consistent with our mechanism.

II. THE CORRELATION FUNCTION OF A SCATTERING MEDIUM WHICH WILL GENERATE DOPPLER-LIKE FREQUENCY SHIFTS

Consider a linearly polarized plane wave, with spectrum $S^{(i)}(\omega)$, incident in a direction specified by a unit vector $\boldsymbol{u}$ on a fluctuating random scattering medium, occupying a volume V. It was recently shown (Wolf and Foley 1989) that, under fairly general conditions, the spectrum of the scattered radiation at a point $\boldsymbol{r} = r\boldsymbol{u}'$ ($|\boldsymbol{u}'| = 1$) in the far zone of the scatterer is given by the following expression, valid within the accuracy of the first-order Born approximation:

$$S^{(\infty)}(r\boldsymbol{u}', \omega') = A\omega'^4 \int_{-\infty}^{\infty} \mathscr{S}\left(\frac{\omega'}{c}\boldsymbol{u}' - \frac{\omega}{c}\boldsymbol{u}, \omega' - \omega; \omega\right) S^{(i)}(\omega) d\omega \,. \tag{1}$$

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function

$$\mathscr{S}(K, \Omega; \omega) = \frac{1}{(2\pi)^4} \int_V d^3R \int_{-\infty}^{\infty} dT\, G(\boldsymbol{R}, T; \omega) e^{-i(\boldsymbol{K} \cdot \boldsymbol{R} - \Omega T)} \quad (2)$$

is the generalized structure function of the medium, being the four-dimensional Fourier transform of the correlation function

$$G(\boldsymbol{R}, T; \omega) = \langle \tilde{\eta}^*(\boldsymbol{r} + \boldsymbol{R}, t + T; \omega)\tilde{\eta}(\boldsymbol{r}, t; \omega)\rangle \quad (3)$$

of the generalized dielectric susceptibility $\tilde{\eta}(\boldsymbol{r}, t; \omega)$ of the scattering medium (Mandel and Wolf 1973). The angular brackets in equation (3) denote the ensemble average. The factor A in equation (1) is given by the expression

$$A = \frac{(2\pi)^3 V \sin^2 \psi}{c^4 r^2}, \quad (4)$$

where ψ is the angle between the electric vector of the incident wave and the direction of scattering and c is the speed of light in vacuum.

As in earlier papers (Wolf 1989a, b), it is convenient to introduce a scattering kernel $\mathscr{K}(\omega', \omega; \boldsymbol{u}', \boldsymbol{u})$ by the formula

$$\mathscr{K}(\omega', \omega; \boldsymbol{u}', \boldsymbol{u}) \equiv \mathscr{S}\left(\frac{\omega'}{c} \boldsymbol{u}' - \frac{\omega}{c} \boldsymbol{u}, \omega' - \omega; \omega\right). \quad (5)$$

Expression (1) for the far-zone spectrum then becomes

$$S^{(\infty)}(r\boldsymbol{u}', \omega') = A\omega'^4 \int_{-\infty}^{\infty} \mathscr{K}(\omega', \omega; \boldsymbol{u}', \boldsymbol{u}) S^{(i)}(\omega) d\omega. \quad (6)$$

Suppose now that the correlation properties of the fluctuating medium are characterized by an anisotropic Gaussian function, viz.,

$$G(\boldsymbol{R}, T; \omega) = G_0 \exp\left[-\left(\frac{X^2}{2\sigma_x^2} + \frac{Y^2}{2\sigma_y^2} + \frac{Z^2}{2\sigma_z^2} + \frac{T^2}{2\tau^2}\right)\right], \quad (7)$$

where σ_x, σ_y, σ_z, τ, and G_0 are positive constants and (X, Y, Z) are the components of the vector $\boldsymbol{R}$ with respect to a suitably chosen set of Cartesian coordinate axes. The correlation function (7) is a natural generalization of those considered by Foley and Wolf (1989) and by Ishimaru (1978). On substituting from equation (7) into equation (2), we readily find that the generalized structure function of the medium is given by

$$\mathscr{S}(K, \Omega; \omega) = \frac{G_0}{(2\pi)^2} \sigma_x \sigma_y \sigma_z \tau \times \exp\left[-\frac{1}{2}\left(\sigma_x^2 K_x^2 + \sigma_y^2 K_y^2 + \sigma_z^2 K_z^2 + \tau^2 \Omega^2\right)\right], \quad (8)$$

where (K_x, K_y, K_z) are the Cartesian components of the vector K with respect to the same coordinate axes.

It follows from equations (8) and (5) that when the correlation function $G(\boldsymbol{R}, T; \omega)$ of the scattering medium is the anisotropic Gaussian distribution (7), the scattering kernel becomes

$$\mathscr{K}(\omega', \omega; \boldsymbol{u}', \boldsymbol{u}) = \frac{G_0}{(2\pi)^2 c} \sigma_x \sigma_y \sigma_z \sigma_\tau \exp\left(-\frac{1}{2}\Delta\right), \quad (9)$$

where

$$\Delta = \frac{1}{c^2}\left[\sigma_x^2(\omega' u_x' - \omega u_x)^2 + \sigma_y^2(\omega' u_y' - \omega u_y)^2 + \sigma_z^2(\omega' u_z' - \omega u_z)^2 + \sigma_\tau^2(\omega' - \omega)^2\right], \quad (10)$$

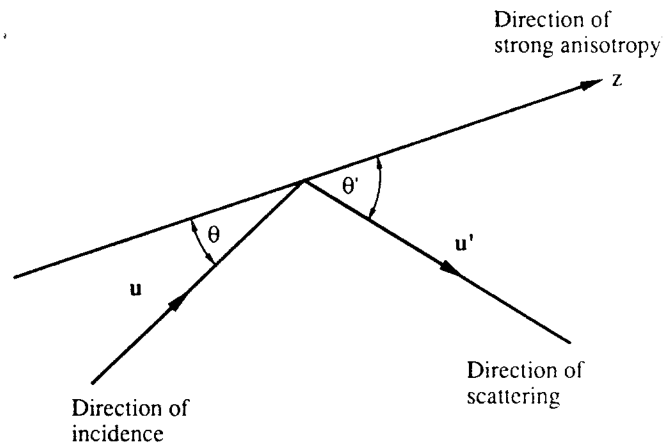


FIG. 1.—Diagram illustrating the significance of the angles θ and θ'

u_x, u_y, u_z and u_x', u_y', u_z' are the components of the unit vectors $\boldsymbol{u}$ and $\boldsymbol{u}'$, respectively, with respect to the same coordinate axes used in connection with equations (7) and (8), and

$$\sigma_\tau \equiv c\tau. \quad (11)$$

Suppose now that the anisotropy of the correlation function $G(\boldsymbol{R}, T; \omega)$ is strong in one particular direction, say in the z-direction, in the sense that

$$\sigma_z \gg \sigma_x, \quad \sigma_z \gg \sigma_y, \quad \text{and } \sigma_z \gg \sigma_\tau. \quad (12)$$

Then, with some additional inequalities being satisfied,[3] equation (10) may be approximated as

$$\Delta \approx \frac{\sigma_z^2}{c^2}(\omega' \cos\theta' - \omega\cos\theta)^2, \quad (13)$$

where θ and θ' are the angles which the unit vectors $\boldsymbol{u}$ (direction of incidence) and $\boldsymbol{u}'$ (direction of scattering) make with the z-axis (see Fig. 1).

With the approximation (13), the scattering kernel (eq. [9]) becomes

$$\mathscr{K}(\omega', \omega; \boldsymbol{u}', \boldsymbol{u}) \approx \frac{\sigma_x \sigma_y \sigma_z \sigma_\tau G_0}{(2\pi)^2} \times \exp\left[-\frac{\sigma_z^2}{2c^2}(\omega' \cos\theta' - \omega\cos\theta)^2\right]. \quad (14)$$

It is convenient to express this formula in the form

$$\mathscr{K}(\omega', \omega; \boldsymbol{u}', \boldsymbol{u}) \approx \frac{\sigma_x \sigma_y \sigma_z \sigma_\tau G_0}{c(2\pi)^2} \exp\left[-\frac{(\alpha\omega' - \omega)^2}{2\Gamma_1^2}\right], \quad (15)$$

where

$$\alpha = \frac{\cos\theta'}{\cos\theta} \quad (16)$$

and

$$\Gamma_1 = \frac{c}{\sigma_z \cos\theta}. \quad (17)$$

[3] If we assume that $\sigma_x = \sigma_y$, then a set of sufficiency, but not necessary, conditions for approximation (13) to hold is $\cos^2\theta' \gg (\sigma_x/\sigma_z)^2$, $\cos^2\theta' \gg (\sigma_\tau/\sigma_z)^2$, $\cos\theta\cos\theta' \gg (\sigma_x/\sigma_z)^2$, $\cos\theta\cos\theta' \gg (\sigma_\tau/\sigma_z)^2$, $\cos^2\theta \gg (\sigma_x/\sigma_z)^2$, and $\cos^2\theta \gg (\sigma_\tau/\sigma_z)^2$.

ession (15) is precisely the form obtained not long ago f 1989a, eq. [16], and 1989b, eq. [6]) for scattering kernels edia which generate frequency shifts that imitate the pler shift in its main features. We will now verify this by cit calculation.

ppose that the spectrum of the incident light consists of a line of Gaussian profile,

$$S^{(i)}(\omega) = I_0 \exp\left[-\frac{(\omega - \omega_0)^2}{2\Gamma_0^2}\right], \tag{18}$$

e ω_0, Γ_0, and I_0 are positive constants. This particular e of the profile is not essential, but it simplifies the calcu-. In order to determine the far-zone spectrum of the scat- light, we must evaluate, according to equation (6), the uct $\mathscr{K}(\omega', \omega; \boldsymbol{u}', \boldsymbol{u})S^{(i)}(\omega)$. We readily find from equations nd (18), after a straightforward but long calculation most performed with the help of the so-called product em for Gaussian functions (Wolf, Foley, and Gori 1989, ndix A), that

$$, \omega; \boldsymbol{u}', \boldsymbol{u})S^{(i)}(\omega) = B \exp\left[-\frac{(\alpha\omega' - \omega_0)^2}{2(\Gamma_0^2 + \Gamma_1^2)}\right] \times \exp\left[-\frac{(\omega - \tilde{\omega})^2}{2\tilde{\Gamma}^2}\right], \tag{19}$$

e

$$B = \frac{1}{c(2\pi)^2} G_0 \sigma_x \sigma_y \sigma_z \sigma_\tau I_0 , \tag{20}$$

$$\tilde{\omega} = \frac{\alpha\omega'\Gamma_0^2 + \omega_0 \Gamma_1^2}{\Gamma_0^2 + \Gamma_1^2}, \tag{21}$$

$$\frac{1}{\tilde{\Gamma}^2} = \frac{1}{\Gamma_0^2} + \frac{1}{\Gamma_1^2}. \tag{22}$$

substituting from equation (19) in the equation (6) and rming the integration, we readily find that

$$S^{(\infty)}(r\boldsymbol{u}', \omega') = C\omega'^4 \exp\left[-\frac{(\omega' - \omega_0')^2}{2(\Gamma_0^2 + \Gamma_1^2)/\alpha^2}\right], \tag{23}$$

e

$$C = \frac{(2\pi)^{3/2} V G_0 I_0 \sigma_x \sigma_y \sigma_z \sigma_\tau \tilde{\Gamma} \sin^2 \psi}{c^5 r^2}, \tag{24}$$

$$\omega_0' = \frac{\omega_0}{|\alpha|}. \tag{25}$$

rmula (23) shows that the spectrum of the scattered field far zone is proportional to the product of the factor ω'^4 Gaussian function. This function is, however, not cen- on the mean frequency ω_0 of the incident light but rather e frequency ω_0' given by equations (25) and (16). This line efore shifted with respect to the spectral line of the incident light (eq. [18]) by the relative amount

$$z \equiv \frac{\omega_0 - \omega_0'}{\omega_0'} = |\alpha| - 1 \tag{26}$$

or, more explicitly, if equation (16) is used, by the relative amount

$$z = \left|\frac{\cos\theta'}{\cos\theta}\right| - 1 . \tag{27}$$

The rms width of this spectral line differs, however, from the rms width Γ_0 of the line of the incident light, being given by

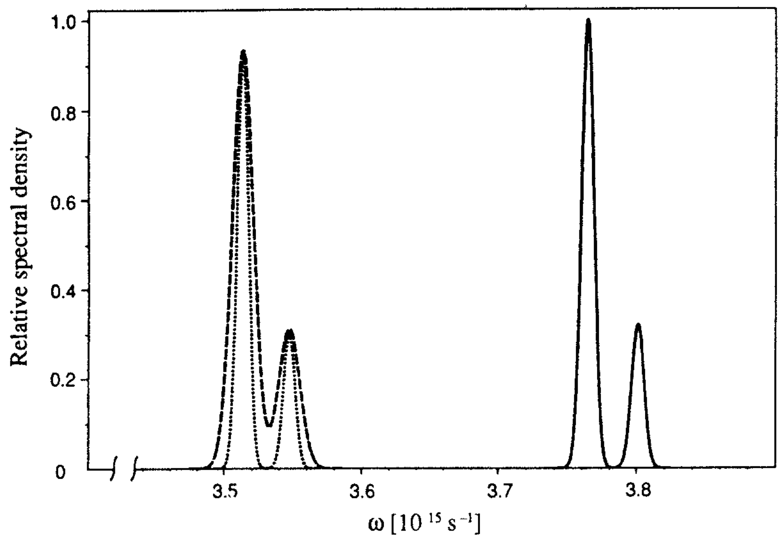
$$\Gamma' = \frac{(\Gamma_0^2 + \Gamma_1^2)^{1/2}}{|\alpha|}. \tag{28}$$

If for a moment we ignore the proportionality factor ω'^4 in equation (23), we see from equation (26) that the *relative frequency shift, which can take any value in the range* $-1 \leq z \leq \infty$, *is independent of the central frequency* ω_0 *of the incident light* and thus imitates the Doppler shift. Evidently the spectral line of the scattered light is redshifted ($z > 0$) when $\theta' < \theta$ and is blueshifted ($z < 0$) when $\theta' > \theta$, with respect to the spectral line of the incident light.

The multiplicative factor ω'^4, just as in the case of Rayleigh scattering, produces a small amount of blueshift which is frequency-dependent; it also produces a frequency-dependent change of the intensity of the line, which has the effect of making the source appear to be bluer. In Figure 2 we illustrate our analysis by an example which shows frequency shifts of a pair of spectral lines due both to this mechanism and to the Doppler effect.

As may be seen from equations (25) and (28), the fractional widths Γ'/ω_0' of the lines in the spectrum of scattered light are different from the fractional widths Γ/ω_0 of the lines in the unshifted spectrum. Such a change does not occur for the Doppler shift in radiation from a source of any state of coherence, as was shown recently (James 1989). However, changes of this ratio occur in other types of correlation-induced spectral shifts (see § I). Further, if more complicated correlation functions than the Gaussian are used, more complicated line profile changes are likely to be produced. However, there may be other physical processes, unrelated to the statistical properties of the fluctuating medium, which can give rise to line irregularities. In this connection it is interesting to recall that Gaskell (1982) discovered systematic differences in redshifts of lines in the same spectrum originating in strongly and weakly ionized emission regions in some quasars; emission lines of highly ionized regions appear to be blueshifted by a relative amount of the order of $z = -0.002$ with respect to the redshift of the narrow spectral lines. In a recent paper (Sulentic 1989) a classification scheme was proposed for the shape and wavelength displacement of the broad emission lines of Hβ with respect to the adjacent forbidden line of O III in active galactic nuclei. This investigation showed that over half of the active galactic nuclei considered had a measurable shift of spectral lines in the broad-line region with respect to those of the narrow-line region. A possibility exists that such irregularities in individual quasar spectra and also discordant redshifts in some quasar-galaxy pairs which appear to be connected may be due to the coherence effect discussed in the present paper.

e have written here $|\alpha|$ rather than α, because when $\alpha < 0$ the negative than the positive frequency part of the spectrum of the incident field is t (see Foley and Wolf 1989, § III, especially eqs. [3.5]–[3.7]).



'wo O III lines ($\lambda = 4959$ Å and $\lambda = 5007$ Å) as seen at rest (*solid line*), Doppler-shifted (*dotted line*), and shifted by the process described in this paper both by a relative amount $z = 0.0714$. The FWHM of both lines was taken as 84 km s^{-1}. The constant C in eq. (23) was chosen so that the height of the ed line is the same as for the Doppler-shifted line. For shifts induced by the correlation mechanism and shown in the figure, $\sigma_z = 50$ μm, $\theta = 30°$, and

III. DISCUSSION

shown that frequency shifts of spectral lines which)oppler effect can be generated by scattering from a :dium which has a physically plausible correlation r the dielectric fluctuations. We emphasize that this cannot be explained either by naive considerations ›hoton fluxes or by radiative transfer or coherent agation. We now briefly examine whether strong s such as are introduced in § II may perhaps be or near the envelopes of some quasars. Our dis- l necessarily be only qualitative, because little infor- ts at present on the detailed structure and geometry regions of quasars.

ired anisotropy is not incompatible with the physi- ters for active galactic nuclei (AGNs) derived from nchrotron radiation model. The relative sizes of the required for our analysis may be estimated from ın AGNs as discussed by Begelman, Blandford, and , in connection with radio galaxies and their jets; by 1986), who justifies placing the quasars as an ener- ;ion of the Seyfert 1 galaxies; and by Osterbrock ;9–88). If the scattering medium is located outside n which the broad-line emission takes place, it may to have similar properties to the narrow-line emit- (NLR). In the vicinity of the NLR, i.e., at a radius of nodeled by Osterbrock, we have a temperature of ectron density $N_e \sim 10^4$ cm^{-3}, and a magnetic field assuming the equipartition of thermal and mag- ıres) of 8×10^{-4} G. This value for the magnetic patible with the value estimated for the nucleus of laxy NGC 6251 by Begelman, Blandford, and Rees

elation function introduced in § II is required to orrelation length, σ_z, significantly larger than the the symmetry has to be strongly broken in one aken to be the z-direction. We assume that this along the jet. (Although Weedman estimates that of all quasars have jets, we assume that jets are characteristic of conditions in these regions.) At a radius of 100 pc it is hard to conceive of structures small compared with the intrinsic size of the "central engine." Thus we take as a characteristic length along the jet a scale comparable to the Schwarzschild radius associated with masses of 10^6–10^9 $M_\odot$, i.e., larger than 2.95×10^{11} cm. As noted by Begelman, Blandford, and Rees (1984), polarization of the observed radio emission from these regions suggests that the magnetic field is oriented predominantly in the direction along the jet. For cyclotron motion of charged particles in this magnetic field we associate characteristic transverse lengths with the gyration radius r_g and a characteristic time with the inverse of the gyration frequency, $2\pi/\omega_g$.

Simple theory for the motion of charged particles in a magnetic field (see, for example, Landau and Lifshitz 1971, § 21) gives the following formulae for these parameters:

$$r_g = \frac{p}{qB} \quad \text{and} \quad \frac{c}{\omega_g} = \frac{\gamma mc}{qB}\,, \tag{29}$$

where p is the relativistic momentum of the particle perpendicular to the magnetic field, q is its charge, m is the particle mass, v is its velocity, and $\gamma = (1 - v^2/c^2)^{-1/2}$. A high degree of anisotropy will exist if r_g and c/ω_g are both less than, say, 3×10^9 cm. The r_g condition therefore implies that the momentum $p \lesssim 750$ MeV/c for both protons and electrons. The ω_g condition will be fulfilled for electrons only if $p \lesssim 1400$ mc. Since most relativistic gases are characterized by a phase-space density proportional to p^β, where β is in the range -4 to -5, the macroscopic properties are dominated by the lower momentum particles. This suggests that the medium is characterized by $p \lesssim mc$ and our conditions are satisfied. Hence it seems that there exist natural anisotropies in some quasar atmospheres appropriate to the scattering mechanism described in § II.

There may be other sources of anisotropy present—for example, those associated with turbulence. However, we wish to mention that anisotropy of the correlation function of the tering medium is not a necessary requirement for gener- g Doppler-like frequency shifts of spectral lines. An mple of an isotropic correlation function which will gener- such shifts was found by James and Wolf (1990) after the pletion of this paper.

ur analysis is rather incomplete for several reasons. In icular, since no adequate knowledge exists at the present about correlations in susceptibility fluctuations in the ity of quasars, our model can only be considered as indic- of some of the possibilities. Further, we only considered tering of an incident plane wave. To draw realistic conclu- s, additional averaging, involving a range of directions of dence, would have to be performed. Moreover, analysis ond the accuracy of the first-order Born approximation, on ch our calculations are based, may be required. Also, we did not treat unscattered radiation. Nevertheless, our analysis clearly indicates that some observed redshifts may contain contributions which arise from the intrinsic properties of the medium surrounding the radiating sources, in addition to those due to the Doppler effect or gravitation. It is thus possible, as we noted in § I, that the long-standing controversy relating to pairs of objects of different redshifts which appear to be physically connected might be resolved by taking into account the correlation mechanism discussed in this paper.

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